Some Useful Information

Greek Alphabet

Α α	Alpha	Ν ν	Nu
Β β	Beta	Ξ ξ	Xi
Γ γ	Gamma	Ο ο	Omicron
Δ δ	Delta	Π π	Pi
Ε ε	Epsilon	Ρ ρ	Rho
Ζ ζ	Zeta	Σ σ	Sigma
Η η	Eta	Τ τ	Tau
Θ θ	Theta	Υ υ	Upsilon
Ι ι	Iota	Φ φ	Phi
Κ κ	Kappa	Χ χ	Chi
Λ λ	Lambda	Ψ ψ	Psi
Μ μ	Mu	Ω ω	Omega

Standard Unit Prefixes

Power of 10	Prefix	Symbol
10^{18}	exa	E
10^{15}	peca	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Conversion Factors

To obtain	*Multiply*	*By*
Inches	Centimeters	0.3937007874
Centimeters	Inches	2.54
Microns	Mils	25.4
Feet	Meters	3.280839895
Yards	Meters	1.093613298
Square Feet	Acres	43560
Miles	Kilometers	0.6213711922
Nautical miles	Miles	0.86897624
Ounces	Grams	$3.52739611922X10^{-2}$
Grams	Ounces	28.34952313
Pounds	Kilograms	2.204622622
Kilograms	Pounds	1.45359237
Gallons	Liters	0.2671729524
Fluid Ounces	Milliliters (cc)	$3.381402270X10^{-2}$
Cubic Inches	Milliliters (cc)	$6.704574409X10^{-2}$
Cubic feet	Cubic Meters	35.31466672
Degrees	Radians	57.2958
Ergs	Foot-pounds	$1.356X10^{7}$
Feet of Water @ 4^{o}C	Atmospheres	33.90
Atmospheres	Pounds/in2	$6.804X10^{-2}$
Inches of Mercury @ 0^{o}C	Pounds/in2	2.036
Joules	BTU	1054.8
Joules	Foot-pounds	1.35582
Kilowatts	Horsepower	0.745712
Kilowatts	Foot-pounds/min	$2.26X10^{-4}$
Watts	BTU/min	17.5796
Degrees Fahrenheit	9/5 (^{o}C) +32	
Degrees Celsius	5/9[(^{o}F)- 32]	

Physical Constants

Speed of light in vacuum	c	$2.997925X10^{8}$ m/s
Gravitational constant	G	$6.670X10^{-11}$ Nm2/kg2
Planck constant	h	$6.6262X10^{-34}$ Js
Rydberg constant	$R_{\bullet}$	$1.0973731X10^{7}$ 1/m
Gas constant	R_0	8.3141 J/deg·mole

Boltzmann constant	k	$1.3806X10^{-23}$ J/deg
Stephan-Boltzmann const.	s	$5.6696X10^{-8}$ $W/m^2 \cdot deg^4$
Euler's Constant	γ	0.57721 56649 01532 86061
Golden Ratio	ϕ	1.61803 39887 49894 84820

Equatorial Radius of Earth.............	6378.388 km, 3963.34 mi statute
Polar Radius of Earth.......................	6356.912 km, 3949.99 mi statute
Gravitational Acceleration...............	$(6.673 \pm 0.003)X10^3$ $cm^3/gm\text{-}s^2$
1 micron..	10^{-4} cm
1 angstrom..	10^{-8} cm
Density of Mercury at 0^oC................	13.5955 g/ml
Density of Water at 3.98^oC.............	1.000000 g/ml
Density of dry air at 0^oC, 760mm...	1.2919 g/liter
Velocity of sound in dry air at 0^oC..	331.36 m/s, 1087.1 ft/s
Heat of fusion of water 100^oC.........	9.71 cal/g

Mathematical Constants

p = 3.14159 26535 89793 23846 26433 83279 50288 41971 69399

e = 2.71828 18284 59045 23536 02874 71352 66249 77572 47093

$\sqrt{2}$ = 1.41424 35623 73095 04880 16887 24209 69807 85696 71875

$\sqrt{3}$ = 1.73205 08075 68877 29352 74463 41505 87236 69428 05253

Arithmetic Progression - An arithmetic progression is a sequence of number such that each number differs from the previous number by a constant amount, called the common difference. Given that a_1 is the first term, a_n is the nth term, d is the common difference, n is the number of terms and s_n is the sum of n terms, we have,

$$a_n = a_1 + (n-1)d,\ s_n = \frac{n}{2}\left[a_1 + a\ \right]$$

$$s_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

Geometric Progression - A geometric progression is a sequence of number such that each number bears a constant ratio, called the common ratio, to the previous number. Given that a_1 is the first term, a_n is the nth term, r is the common ratio, n is the number of terms and s_n is the sum of n terms, we have,

$$a_n = a_1 r^{n-1},\ s_n = a_1 \frac{1-r^n}{1-r}$$

$$= a_1 \frac{r^n - 1}{r-1} \quad for\ r \neq 1$$

$$= \frac{a_1 - ra_n}{1-r}$$

$$= \frac{ra_n - a_1}{r-1}$$

Harmonic Progression - A sequence of number whose reciprocals form an arithmetic progression is called an harmonic progression. We have,

$$\frac{1}{a_1}, \frac{1}{a_1+d}, \frac{1}{a_1+2d}, \ldots, \frac{1}{a_1+(n-1)d}, \ldots,$$

where,

$$\frac{1}{a_n} = \frac{1}{a_1+(n-1)d}$$

forms an harmonic progression.

Trignometric Relationships

Express... / in terms of...	sin	cos	tan
$\sin x$	-	$\sqrt{(1 - \cos^2 x)}$	$\frac{\tan x}{\sqrt{(1+\tan^2 x)}}$
$\cos x$	$\sqrt{(1 - \sin^2 x)}$	-	$\frac{1}{\sqrt{(1+\tan^2 x)}}$
tan x	$\frac{\sin x}{\sqrt{(1 - \sin^2 x)}}$	$\frac{\sqrt{(1 - \cos^2 x)}}{\cos x}$	-

Functions of Some Special Angles

Angle	sin	cos	tan
0	0	1	0
$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	∞
$2\pi/3$	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$
$3\pi/4$	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1
$5\pi/6$	$1/2$	$-\sqrt{3}/2$	$-\sqrt{3}/3$
π	0	-1	0
$3\pi/2$	-1	0	∞

DeMoivre's Theorem for Polar Forms

$$(\cos q + i\sin q)^n = \cos nq + i\sin nq$$

Euler's Formula

$$e^{iq} = \cos q + i\sin q$$

Sum and Difference Relations

$$\sin(a+b)=\sin a\cos b+\cos a\sin b$$
$$\sin(a-b)=\sin a\cos b-\cos a\sin b$$
$$\cos(a+b)=\cos a\cos b+\sin a\sin b$$
$$\cos(a-b)=\cos a\cos b+\sin a\sin b$$
$$\sin(a+b)\sin(a-b)=\sin^2 a-\sin^2 b$$
$$\sin(a+b)\sin(a-b)=\cos^2 a-\cos^2 b$$
$$\cos(a+b)\cos(a-b)=\cos^2 a-\sin^2 b$$
$$\cos(a+b)\cos(a-b)=\cos^2 b-\sin^2 a$$

$$\sin a+\sin b=2\sin\frac{1}{2}(a+b)\cos\frac{1}{2}(a-b)$$
$$\sin a-\sin b=2\cos\frac{1}{2}(a+b)\sin\frac{1}{2}(a-b)$$
$$\cos a+\cos b=2\cos\frac{1}{2}(a+b)\cos\frac{1}{2}(a-b)$$
$$\cos a-\cos b=-2\sin\frac{1}{2}(a+b)\sin\frac{1}{2}(a-b)$$

Product Relations

$$\sin a\sin b=1/2\cos(a-b)-1/2\cos(a+b)$$
$$\sin a\cos b=1/2\sin(a+b)+1/2\sin(a-b)$$
$$\cos a\sin b=1/2\sin(a+b)-1/2\sin(a-b)$$
$$\cos a\cos b=1/2\cos(a-b)+1/2\cos(a+b)$$

Power and Exponential Relations

$$\sin^2 a = \frac{1}{2}(1 - \cos 2a), \quad \cos^2 a = \frac{1}{2}(1 + \cos 2a)$$

$$e^{ia} = \cos a + i \sin a \quad where\ i = \sqrt{-1}$$

$$\sin a = \frac{e^{ia} - e^{-ia}}{2i}, \quad \cos a = \frac{e^{ia} + e^{-ia}}{2i}$$

$$\tan a = -i\left(\frac{e^{ia} - e^{-ia}}{e^{ia} + e^{-ia}}\right) = -i\left(\frac{e^{2ia} - 1}{e^{2ia} + 1}\right)$$

Plane Triangle Formulas

In the following, A, B, and C denote the angles of any plane triangle, a, b, and c are the corresponding opposite sides.

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Law of Tangents

$$\frac{a - b}{a + b} = \frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)}$$

Newton's Formula

$$\frac{a+b}{c}=\frac{\cos\frac{1}{2}(A-B)}{\sin\frac{1}{2}C}$$

Mollweide's Formula

$$\frac{a-b}{c}=\frac{\sin\frac{1}{2}(A-B)}{\cos\frac{1}{2}C}$$

Area

$$K=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B=\frac{1}{2}ab\sin C$$

$$K=\frac{a^2\sin B\sin C}{2\sin A}=\frac{b^2\sin C\sin A}{2\sin B}=\frac{c^2\sin A\sin B}{2\sin C}$$

Combinatorics

Permutations - The number of permutations *P(n,m)* of *n* distinct items taken *m* at a time (i.e. their order matters) is given by,

$$P(n,m)=\frac{n!}{(n-m)!}=n(n-1)(n-2)\cdots(n-m+1)$$

Combinations - The number of combinations C(n,m) of n distinct items taken m at a time (i.e. their order does not matter) is given by,

$$\binom{n}{m}=C(n,m)=\frac{n!}{m!(n-m)!}=\frac{P(n,r)}{m!}$$

The Pascal's Triangle is related to C(n,m) by the recurrence relation,

$$\begin{array}{c} 1 \\ 1\ \ 1 \\ 1\ \ 2\ \ 1 \\ 1\ \ 3\ \ 3\ \ 1 \\ 1\ \ 4\ \ 6\ \ 4\ \ 1 \\ 1\ \ 5\ \ 10\ \ 10\ \ 5\ \ 1 \\ 1\ \ 6\ \ 15\ \ 20\ \ 15\ \ 6\ \ 1 \\ \vdots \qquad \vdots \end{array}$$

$$\binom{n+1}{m+1} = \binom{n}{m} + \binom{n}{m+1}$$

Given *n* total objects partitioned into *m* groups such that the members of a particular group are identical while a member of one group is distinct from the member of another group, we may determine the number of arrangements of these *n* objects as,

$$P(n; r_1, r_2, \cdots, r_m) = \frac{n!}{r_1! r_2! \cdots r_m!}$$

where there are r_i objects in group *i*.

The number of combinations of C(n,m) is prevalent in applied mathematics. See, for example, *Series Expansions* in this appendix.

Series Expansions

Integer Series

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2 (n+1)^2}{4}$$

Polynomial Expansions

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y^1 + \binom{n}{2}x^{n-2}y^2 + \cdots + \binom{n}{n-1}x^1y^{n-1} + \binom{n}{n}y^n$$

$$\frac{1-x^{n+1}}{1-x} = 1 + x + x^2 + \cdots + x^n$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots$$

Binomial Identities

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^2$$

$$\binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \cdots + \binom{n+r}{r} = \binom{n+r+1}{r}$$

$$\sum_{k=0}^{r}\binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$$

$$\sum_{k=0}^{m}\binom{m}{k}\binom{n}{r-k} = \binom{m+n}{m+r}$$

Exponential Series

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$a^x = 1 + x\log_e a + \frac{(x\log_e a)^2}{2!} + \frac{(x\log_e a)^3}{3!} + \cdots$$

Trigonometric Series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

Taylor Series

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a) + \cdots + \frac{(x-a)^{n-1}}{(n-1)!} f^{(n-1)}(a) + R_n$$

where

$$R_n = \frac{f^{(n)}\left[a + ø \cdot (x-a)\right]}{n!}(x-a)^n \quad \text{for some value of } ø,\ 0 < ø < 1.$$

MaClaurin Series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \cdots + \frac{x^{n-1}}{(n-1)!} f^{(n-1)}(0) + R_n$$

where

$$R_n = \frac{f^{(n)}[øx]}{n!} x^n \quad \text{for some value of } ø,\ 0 < ø < 1.$$

Analytic Geometry

Distance between two points (x_1,y_1) and (x_2,y_2) in a cartesian coordinate system.

$$\sqrt{(x_2-x_1)+(y_2-y_1)}$$

The midpoint between two points (x_1,y_1) and (x_2,y_2) in a cartesian coordinate system.

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

The slope of the line connecting two points (x_1,y_1) and (x_2,y_2) in a car-tesian coordinate system.

$$m = \tan ø = \frac{y_2-y_1}{x_2-x_1}$$

Three points (x_1,y_1) and (x_2,y_2) and (x_3,y_3) lie in a straight line only if the determinant shown is zero.

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

The equation for the line connecting two points (x_1,y_1) and (x_2,y_2) in a cartesian coordinate system.

$$\frac{y-y_1}{x-x_1}=\frac{y-y_2}{x-x_2}$$

Equation for circle with radius r and center at position (x_c,y_c).

$$(x-x_c)^2+(y-y_c)^2 = r^2$$

The equation of a line determined by a point (x_1,y_1,z_1) and a direction vector **a** = (a_1,a_2,a_3).

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{a_2}=\frac{z-z_1}{a_{31}}$$

The distance d of a point (x_0,y_0,z_0) from the line

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{a_2}=\frac{z-z_1}{a_3}$$

$$d = \frac{|\mathbf{u}\times\mathbf{a}|}{|\mathbf{a}|};\ \mathbf{a}=(a_1,a_2,a_3)$$

$$\mathbf{u} = (x_0-x_1, y_0-y_1, z_0-z_1)$$

The distance d between two skew lines $\mathbf{r}_1+\mathbf{a}t$ and $\mathbf{r}_2+\mathbf{b}t'$.

$$d=\frac{\left|[(\mathbf{r}_2-\mathbf{r}_1)\mathbf{a}\mathbf{b}]\right|}{|\mathbf{a}\times\mathbf{b}|}$$

Coordinate Transformations

Translation - The coordinate values of points in a Rectangular coordinate system to a new coordinate system whose origin is at a point (x_c,y_c) in the original system.

$$x_{new} = x_{old} - x_c$$

$$y_{new} = y_{old} - y_c$$

Rotation - The coordinate values of points in a Rectangular coordinate system to a new coordinate system whose axes make an angle ø with the original coordinate system axes.

$$x_{new} = x_{old} \cos ø - y_{old} \sin ø$$

$$y_{new} = y_{old} \cos ø + x_{old} \sin ø$$

Polar - The transformations between a point (x,y) in Rectangular coordinates and Polar coordinates r, ø is given by,

$$x = r \cos ø \qquad r = \sqrt{x^2 + y^2}$$

$$y = r \sin ø \qquad ø = \arctan \frac{y}{x}$$

Spherical - The transformations between Spherical (r, ø, µ) and Rectangular (x, y, z) coordinates, where µ is the angle of the radial vector r measured from the z axis and ø is the azimuthal angle measured counter-clockwise from the x axis in the x-y plane.

$$x = r \sin ø \cos \mu \quad y = r \sin ø \sin \mu \quad z = r \cos ø$$

$$r = \sqrt{x^2 + y^2 + z^2} \quad \mu = \arccos \frac{z}{r} \quad ø = \arctan \frac{y}{x}$$

Direction Cosines - The angles between each axis of a Rectangular coordinate system and a line connecting the points (x_1,y_1.z_1) and (x_2,y_2.z_2)are called the direction angles, and are described by,

$$\cos a = \frac{x_2 - x_1}{d} \quad \cos b = \frac{y_2 - y_1}{d} \quad \cos g = \frac{z_2 - z_1}{d}$$

$$where \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$and \qquad \cos^2 a + \cos^2 b + \cos^2 g = 1$$

Basic Statistical Concepts

Arithmetic Mean - The mean or average of n measurements of a random variable is calculated by,

$$\frac{1}{n}\sum_{i=1}^{n} x_i$$

Harmonic Mean - Useful for computing average ratios such as average miles/hr. The harmonic mean of a and b is,

$$\frac{1}{\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)}=\frac{2ab}{a+b}$$

Geometric Mean - Useful for computing average rates and when the data are distributed exponen-tially, geometric mean of a & b is,

$$\sqrt{ab}$$

Median - The middle value in a set of n measurements. Given a sequence of values such that,

$$x_1 \le x_2 \le \cdots x_n$$

$$x_{med} = x_{(n+1)/2} \qquad \textit{for n odd,}$$

$$x_{med} = \frac{1}{2}\left(x_{n/2} + x_{n/2+1}\right) \qquad \textit{for n even}$$

Mode - The value that occurs most often in a set of n measurements.

Mean Deviation -The absolute value of the difference between a particular meausurement and the average of all the measurements.

$$\frac{\sum_{i=1}^{n}\left|\bar{x} - x_i\right|}{n}$$

Standard Deviation - The square root of the average value of the square of the individual deviations.

$$s = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}} \quad ; \quad s^2 = variance$$

Moments - The *rth* moment about the mean is given by,

$$m_r = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^r$$

Skewness - The coefficient of skew-ness in terms of the *2nd* and *3rd* moments about the mean of the sample,

$$\frac{m_3}{\left(m_2\right)^{3/2}}$$

Kurtosis - In terms of the *2nd* and *4th* moments about the mean,

$$\frac{m_4}{\left(m_2\right)^2}$$

Some Important Probability Distributions

Binomial Distribution - A Binomial Experiment is one that consists of repeated trials with two possible outcomes, such that the probability of either outcome in a trial is constant and independent of the other trials. If a binomial trial can result in a success with probability p then the probability of failure is q=(1-p) and the probability that exactly m out of n trials will result in a success is,

$$b(m,n,p)=\binom{n}{m}p^{m}(1-p)^{n-m} \qquad \mathit{for}\ m=0,1,2,\cdots,n$$

Hypergeometric Distribution - Given that n items are to be selected from a collection of N items, in which k have been labeled as successes and N-k have been labeled as failures, we may determine the probability that exactly m out of the n selected items are labeled successes by,

$$h(m,N,n,k)=\frac{\binom{k}{m}\binom{N-k}{n-m}}{\binom{N}{n}} \qquad \mathit{for}\ m=0,1,2,\cdots,n$$

Poisson Distribution - The Poisson distribution relates the probability of a specific number of occurances of an event within a specified time or region to the average number of occurances of that event in the same time or region. The probability distribution representing the number of outcomes occuring in a given time interval or specified region is,

$$p(n,\mu)=\frac{e^{-\mu}\mu^{n}}{n!} \qquad \mathit{for}\ n=0,1,2,\cdots$$

where µ is the average number of occurences in the specified time interval or region.

Normal (Gaussian) Distribution - When repeated measurements of a random variable with a constant mean µ are made, the distribution of values is represented by,

$$g(x,m,s)=\frac{1}{\sqrt{2\pi}\,s}\,e^{-\frac{1}{2}\left[\frac{(x-m)}{s}\right]^{2}} \qquad \mathit{for\ all}\ x.$$

where σ is the standard deviation of the random variable.

Fourier Series

A function f(x) is periodic with period T if f(x) = f(x+T) for all x. If f(x) is a bounded periodic function of period T with a finite number of step discontinuities and a finite number of maxima and minima then it may be represented by,

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{2n\pi x}{T}+b_n\sin\frac{2n\pi x}{T}\right)$$

called the Fourier series. The values of a_n and b_n are defined as,

$$a_n = \frac{2}{T}\int_0^T f(x)\cos\frac{2n\pi x}{T}\,dx$$

$$b_n = \frac{2}{T}\int_0^T f(x)\sin\frac{2n\pi x}{T}\,dx,\ \ n = 1, 2, 3, \cdots$$

With the inclusion of a phase term $ø_n$, the function f(x) can be represented as,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} c_n \cos\left(\frac{2n\pi x}{T} + ø_n\right)$$

where

$$a_n = c_n \cos ø_n,\ b_n = -c_n \sin ø_n,\ c_n = \sqrt{a_n^2 + b_n^2},\ ø_n = \arctan\left(-\frac{b_n}{a_n}\right)$$

The Fourier series may also be written in exponential form by applying Euler's function to the sine and cosine terms.

$$f(x) = \frac{1}{2}\sum_{n=-\infty}^{\infty} c_n e^{iw_n x}$$

where

$$c_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)e^{-iw_n x}dx,\ \ n = 0, \pm1, \pm2, \pm3, \cdots$$

with

$$w_n = \frac{2n\pi}{T},\ \ n = 0, \pm1, \pm2, \pm3, \cdots$$

The set of coefficients c_n is often referred to as the Fourier spectrum of f(x).

Fourier Transforms

Given a piecewise continuous function f(x) over a finite interval x=[0,¹], then finite Fourier cosine transform of f(x) is,

$$F_c(n) = \int_0^{\pi} f(x)\cos nx\,dx \quad for\ n = 0, 1, 2, \cdots$$

with the inverse transform given by,

$$\bar{f}(x) = \frac{1}{\pi}F_c(0) + \frac{2}{\pi}\sum_{n=1}^{\infty} F_c(n)\cos nx \quad for\ 0 < x < \pi$$

In an analogous manner we may define the Fouriner sine transform of f(x) is,

$$F_s(n) = \int_0^{\pi} f(x)\sin nx\,dx \quad for\ n = 1, 2, 3, \cdots$$

with the inverse transform given by,

$$\bar{f}(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} F_s(n) \sin nx \quad for\ 0 < x < \pi$$

In its exponential form, the Fourier transform of f(x) is,

$$F(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{iax}\, dx$$

with the inverse transform given by,

$$\bar{f}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(a) e^{-iax}\, da.$$

If f(x) <=> F(ø) and g(x) <=> F(ø) (where <=> is read as "is the Fourier transform of"), then the following properties hold,

linearity f(x) + g(x) <=> F(ø) + G(ø)
symmetry F(x) <=> f(-ø)
time scaling f(kx) <=> 1/|k| F(f/k)
frequencyscaling 1/|k|f(x/k)<=>F(kø)
time shift $f(x-x_0)$ <=> $F(x)e^{-i2^{1}øxo}$
frequency shift $f(x)e^{i2^{1}xøo}$ <=> $F(ø-ø_0)$